ON THE PRINCIPLE OF MINIMUM POWER IN AN ARC DISCHARGE

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It follows from considerations of the relationship between the power dissipated in an arc and the entropy developed that in the case of stationary states which are characterized by the minimum of entropy generation, the power dissipated in an arc discharge does not reach its minimum [1]. A class of states was found which can be used for calculating the conditions in the arc discharge with the aid of a variational technique based upon the formal variation of power.

The Steenbeck minimum principle [2] is known in the theory of hot arcs: at a given current I of the arc and at a fixed temperature T_k at the walls of the discharge chamber, we have

$$\delta E = 0 \tag{1}$$

where E denotes the electric field strength in the arc (the field strength is constant in the volume of a d.c. arc channel with cylindrical symmetry).

It was attempted in [1] to show that the variation condition (1) results from the thermodynamic principle of minimum generation of entropy θ:

$$\delta\theta = 0 \tag{2}$$

To this end, the relation between the generation of entropy θ and the power N dissipated in the arc was used (Guy–Stodall Law):

$$N = T_k\theta \tag{3}$$

Variation of Eq. (3) with proper regard for Eq. (2) led to the conclusion that the dissipation is minimal in a stationary arc:

$$\delta N = 0 \tag{4}$$

When we now use the relation between the dissipation and the field strength of the electric field in the arc

$$N = IEl \tag{5}$$

where l denotes the fixed length of the discharge, we obtain condition (1) from Eqs. (4) and (5).

This procedure was based upon the validity of the variation of Eq. (3). Obviously, for this operation to be admissible, Eq. (3) must be interpreted as identity of two functionals which are given on some finite set of functions.

In the thermodynamics of irreversible processes (see, e.g. [3]), the generation of the entropy θ is considered a functional which is given on a set of temperature distributions T(x, y, z) or T(r) when the system has cylindrical symmetry. When external boundary conditions of the form

$$\begin{aligned} &T(R) = T_k \text{ (constant cooling of walls)} \\ &dT/dr|_{r=0} = 0 \text{ (symmetry condition)} \end{aligned} \tag{6}$$

are imposed upon a thermodynamic system which is a cylinder of radius R, we can assume an innumerable set of temperature distributions T(r) which satisfy Eq. (6). The minimum principle of entropy generation

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states that in the stationary state, the temperature distribution $T_S(r)$ for which the generation of the entropy θ (considered to be a functional) has an extremum will be really observed. This is true, because the Euler–Lagrange equation of the variational problem of determining the extremum is the stationary equation of the energy balance (the Ellenbaas–Heller equation in the case of an arc). Any other distribution T(r) which satisfies Eq. (6) and is close to $T_S(r)$ corresponds to some nonstationary state of the thermodynamic system, with the system gradually transforming into the stationary state $T_S(r)$ in the course of time.

This means that the region in which θ is defined is equal to any distribution T(r), i.e., nonstationary as well as stationary distributions; the generation of the entropy θ has an extremum in the stationary state. The validity of Eq. (3) was proved in [1] only for the stationary state, and therefore, Eq. (3) so far only states the fact that the two functionals are equal for a single temperature distribution $T_S(r)$. In view of what has been said above, the variation of Eq. (3) does not make sense. It was shown in [4] that the conclusions of [1] are wrong.

In order to use the extremum properties of entropy generation for the purpose of drawing conclusions on the behavior of the dissipated power N in the stationary state, we must consider the identity relation between N and θ, which is also valid for the nonstationary states of the arc. In order to determine this relation, we must use an explicit expression for the generation of the entropy θ. When only thermal conductivity and electric conductivity are irreversible processes in an arc with cylindrical symmetry, then, in accordance with [2], we have

$$\theta = \int\left[-\frac{(\mathbf{W}\cdot\operatorname{grad} T)}{T^2} + \frac{(\mathbf{j}\cdot\mathbf{E})}{T}\right] dV \tag{7}$$

where **W** denotes the vector of the heat flux density; **j** denotes the electric current density; and the integration is performed over the entire volume of the system.

For this case, the nonstationary equation of the energy balance assumes the form

$$\rho c \frac{\partial T}{\partial t} = -\operatorname{div}\mathbf{W} + (\mathbf{j}\cdot\mathbf{E}) \tag{8}$$

where ρ denotes the density, and c, the specific heat of the gas of the arc. We can use Eq. (8) to rewrite Eq. (7) as follows:

$$\theta = \int\left[-\frac{\mathbf{W}\operatorname{grad} T}{T^2} + \frac{\operatorname{div}\mathbf{W}}{T}\right] dV + \int\left(\rho c\frac{\partial T}{\partial t}\frac{1}{T}\right) dV \tag{9}$$

Since the expression under the first integral of Eq. (9) is equivalent to div(W/T), we can use the Gauss theorem and the condition that the temperature is constant at the boundary of the arc. We obtain

$$\theta = \frac{1}{T_k}\int(\operatorname{div}\mathbf{W})\, dV + \int\left(\rho c\frac{\partial T}{\partial t}\frac{1}{T}\right) dV \tag{10}$$

After eliminating div W from Eq. (10) with the aid of Eq. (8) and taking into account that

$$N = \int(\mathbf{j}\cdot\mathbf{E})\, dV \tag{11}$$

we finally obtain

$$N = T_k\theta + \int \rho c\frac{\partial T}{\partial t}\left(1 - \frac{T_k}{T}\right) dV\ ; \tag{12}$$

Eq. (12) is a generalization of Eq. (3) to the case of arbitrary nonstationary states. Naturally, in the case of a stationary state, we have

$$(\partial T/\partial t)_s \equiv 0 \tag{13}$$

in the entire volume of the system, and Eq. (12) becomes Eq. (3).

The following detail must be mentioned. Though the generation of the entropy θ is considered a functional which is given on the set of all possible temperature distributions (among them nonstationary temperature distributions), θ depends explicitly only upon the characteristics of the states proper but is independent of the rate of change of these states in the course of time. It follows from Eq. (12) that the dissipation N depends upon the distributions T(r) proper, as well as upon the spatial distribution of the derivatives $\partial T/\partial t$ (r) which can have any form [the derivatives must satisfy Eq. (8), but this was taken into account in the derivation of Eq. (12)].

By varying condition (12) with proper regard for Eqs. (2) and (13), we obtain

$$(\delta N)_s = \int \rho_s c_s \left(1 - \frac{T_k}{T}\right) \frac{\partial T}{\partial t} dV \tag{14}$$

Since it is always possible to imagine a state of the system close to the stationary state for which the derivative $\partial T/\partial t\,(r)$ conserves its sign for all r, we have in particular for the corresponding variation $\delta(\partial T/\partial t)$

$$(\delta N)_s \neq 0 \tag{15}$$

i.e., the stationary nonequilibrium distribution of the temperature does not provide an extremum of the power N to be dissipated in the stationary state, though the generation of the entropy θ has an extremum in this state.

Nevertheless, Eq. (12) allows an implicit determination of the class of states for which Eq. (3) holds as an identity. It follows from Eq. (12) that the corresponding temperature distributions must satisfy the condition

$$\int \rho c \frac{\partial T}{\partial t} \left(1 - \frac{T_k}{T}\right) dV = 0 \tag{16}$$

which, in view of Eq. (8), can be rewritten in the form

$$\int [(\mathbf{j}\cdot\mathbf{E}) - \operatorname{div} W] \left(1 - \frac{T_k}{T}\right) dV = 0 \tag{17}$$

Thus, it has been shown that even when the principle of minimum generation of entropy holds, the power dissipated in the stationary state of the arc is not a minimum compared with the power dissipated in nonstationary states which are close to the stationary state. This conclusion is contrary to that of [1]. However, since Eq. (2) is for certain restrictions equivalent to the Ellenbaas–Heller equation, a variational approach to the calculation of the stationary conditions of an arc discharge is possible when the principle of the minimum of entropy generation is employed. The fact that the power dissipated has an extremum value can also be employed in the form of Eq. (4), though this is not possible for any temperature distribution but only for temperature distributions satisfying condition (17). In particular, it is easy to show that temperature distributions corresponding to the approximation of the "channel" model of the arc column (see, e.g. [2]) satisfy condition (17).

LITERATURE CITED

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